**Appendix V**

The Outlier Tests according to Grubbs (F.E.Grubbs and G.Beck, Technometrics, 14, (1972), 847.)

1. Calculation of the Test Value $r''_{lower}$ and $r''_{upper}$

   Arrange the individual laboratory averages $\bullet_i$ in ascending order:

   $\bullet_{(1)}$ = smallest laboratory value

   $\bullet_{(n)}$ = largest laboratory value.

   Calculate the total mean, $\bullet$ and standard variation, s.

   Calculate the difference between:
   the total mean and the smallest laboratory average value

   $$= \bullet - \bullet_{(1)}$$ and

   the total mean and the largest laboratory average value

   $$= \bullet_{(n)} - \bullet$$

   Compare the two differences, and with the largest difference calculate:

   $$r''_{lower} = \frac{\bullet - \bullet_{(1)}}{s}$$

   $$r''_{upper} = \frac{\bullet_{(n)} - \bullet}{s}$$

   These test values $r''_{lower}$ and $r''_{upper}$ are compared with the corresponding tabulated critical values $r_{\bullet,n}$ (see Tables)

2. Evaluation

   If
   $r''_{lower} > r_{\bullet,n}$
   or
   $r''_{upper} > r_{\bullet,n}$
   then the extreme value checked, $\bullet_{(1)}$ or $\bullet_{(n)}$ is an outlier at the probability value of $\bullet$. It is recommended that a probability value of $\bullet = 0.01$ should be taken for the testing of the total values.

Table

Critical values $r_{\alpha;n}$ for the Grubbs outlier test

Two-sided test

| n \ α | 0.10 | 0.05 | 0.02 | 0.01 |
|---|---|---|---|---|
| 3 | 1.153 | 1.155 | 1.155 | 1.555 |
| 4 | 1.463 | 1.481 | 1.492 | 1.496 |
| 5 | 1.672 | 1.715 | 1.749 | 1.764 |
| 6 | 1.822 | 1.887 | 1.944 | 1.973 |
| 7 | 1.938 | 2.020 | 2.097 | 2.139 |
| 8 | 2.032 | 2.12'6 | 2.221 | 2.274 |
| 9 | 2.110 | 2.215 | 2.323 | 2.387 |
| 10 | 2.176 | 2.290 | 2.410 | 2.482 |
| 11 | 2.234 | 2.355 | 2.485 | 2.564 |
| 12 | 2.285 | 2.412 | 2.550 | 2.636 |
| 13 | 2.331 | 2.462 | 2.607 | 2.699 |
| 14 | 2.371 | 2.507 | 2.659 | 2.755 |
| 15 | 2.409 | 2.549 | 2.705 | 2.806 |
| 16 | 2.443 | 2.585 | 2.747 | 2.852 |
| 17 | 2.475 | 2.620 | 2.785 | 2.894 |
| 18 | 2.504 | 2.651 | 2.821 | 2.932 |
| 19 | 2.532 | 2.681 | 2.854 | 2.968 |
| 20 | 2.557 | 2.709 | 2.884 | 3.001 |
| 21 | 2.580 | 2.733 | 2.912 | 3.031 |
| 22 | 2.603 | 2.758 | 2.939- | 3.060 |
| 23 | 2.624 | 2.781 | 2.963 | 3.087 |
| 24 | 2.644 | 2.802 | 2.987 | 3.112 |
| 25 | 2.663 | 2.822 | 3.009 | 3.135 |
| 26 | 2.681 | 2.841 | 3.029 | 3.157 |
| 27 | 2.698 | 2.859 | 3.049 | 3.178 |
| 28 | 2.714 | 2.876 | 3.068 | 3.199 |
| 29 | 2.730 | 2.893 | 3.085 | 3.218 |
| 30 | 2.745 | 2.908 | 3.103 | 3.236 |
| n \ α/2 | 0.05 | 0.025 | 0.01 | 0.005 |

One-sided test

Modified table according to F.E. Grubbs and G. Beck
Technometrics Vol. 14 (1972) pp. 847 et seq.

Table continued

Critical values $r_{\alpha;n}$ for the GRUBBS outlier test

Two-sided test

| n \ α | 0.10 | 0.05 | 0.02 | 0.01 |
|---|---|---|---|---|
| 31 | 2.759 | 2.924 | 3.119 | 3.253 |
| 32 | 2.773 | 2.938 | 3.135 | 3.270 |
| 33 | 2.786 | 2.952 | 3.150 | 3.286 |
| 34 | 2.799 | 2.965 | 3.164 | 3.301 |
| 35 | 2.811 | 2.979 | 3.178 | 3.316 |
| 36 | 2.823 | 2.991 | 3.191 | 3.330 |
| 37 | 2.835 | 3.003 | 3.204 | 3.343 |
| 38 | 2.846 | 3.014 | 3.216 | 3.356 |
| 39 | 2.857 | 3.025 | 3.228 | 3.369 |
| 40 | 2.866 | 3.036 | 3.240 | 3.381 |
| 42 | 2.887 | 3.057 | 3.261 | 3.404 |
| 44 | 2.905 | 3.075 | 3.282 | 3.425 |
| 46 | 2.923 | 3.094 | 3.302 | 3.445 |
| 48 | 2.940 | 3.111 | 3.319 | 3.464 |
| 50 | 2.956 | 3.128 | 3.336 | 3.483 |
| 52 | 2.971 | 3.143 | 3.353 | 3.500 |
| 54 | 2.986 | 3.158 | 3.368 | 3.516 |
| 56 | 3.000 | 3.172 | 3.383 | 3.531 |
| 58 | 3.013 | 3.186 | 3.397 | 3.546 |
| 60 | 3.025 | 3.199 | 3.411 | 3.560 |
| 65 | 3.055 | 3.230 | 3.442 | 3.592 |
| 70 | 3.082 | 3.257 | 3.471 | 3.622 |
| 75 | 3.107 | 3.282 | 3.396 | 3.648 |
| 80 | 3.130 | 3.305 | 3.521 | 3.673 |
| 85 | 3.151 | 3.327 | 3.543 | 3.695 |
| 90 | 3.171 | 3.347 | 3.563 | 3.716 |
| 95 | 3.189 | 3.365 | 3.582 | 3.736 |
| 100 | 3.207 | 3.383 | 3.600 | 3.754 |

| n \ α/2 | 0.05 | 0.025 | 0.01 | 0.005 |
|---|---|---|---|---|

One-sided test

Modified table according to F.E. Grubbs and G. Beck
Technometrics Vol. 14 (1972) pp. 847 et seq.